T_o = maximum temperature to which char has been exposed previous to gasification in IGT kinetic scheme
T_p = particle temperature
T_{P2} = feed particle temperature
T_{PS} = solid external surface temperature
T_w = temperature of internal walls at the bottom of gasifier
t = time
w_{AF} = ash mass fraction in solid feed
w_i = mass fraction of i^{th} gaseous component
X^* = solid conversion at the transition between zones
x = axial coordinate in reactor
x^* = location of the transition between combustion and gasification zones
X_f = final conversion

Greek Letters

ΔH_i = enthalpy change for the i^{th} reaction
δ_H = ash-layer thickness whose resistance to heat transfer is equivalent to that of the film surrounding the particle
ϵ_b = bed porosity
ξ = sum of water and carbon dioxide concentrations
ρ_a = mass of ash per unit particle volume
ρ_c = mass of fixed carbon per unit particle volume
ρ_s = particle density
ρ_{SF} = particle density at feed

Subscript

B = condition in the bulk phase
c = condition at the core surface
1 = condition at the transition between gasification and combustion zone
2 = condition at the top of the reactor

Superscript

o = condition at the gas inlet
c = condition at the core surface

LITERATURE CITED

Arri, L. E., and Neal R. Amundson, "An Analytical Study of Single Particle Char Gasification," *AIChE J.*, (in press).
Beverage, G. S. G., and P. J. Goldie, "Analysis of Non-Isothermal Moving Beds for Non-Catalytic Solid-Gas Reactions," *Chem. Eng. Sci.*, 23, 913 (1968).
Elgin, D. C., and H. R. Perks, "Trials of American Coals in a Lurgie Pressure Gasification Plant at Wesefield, Scotland," *I.G.T. Symposium Papers*, p. 127 (1973).
Elgin, D. C., "Results of Trials of American Coals in a Lurgi Pressure Gasification Plant at Westfield, Scotland," Proceedings of 6th Synthetic Pipeline Gas Symposium, p. 247 (1974).
Hebden, D., "High Pressure Gasification under Slagging Conditions," Proceedings of 7th Synthetic Pipeline Gas Symposium, p. 385 (1975).
Hoogendorn, J. C., "Gas From Coal with Lurgi Gasification at Sasol," *I.G.T. Symposium Papers*, p. 111 (1973).
Ishida, M., and C. Y. Wen, "Analysis of Non-Isothermal Moving Bed for Non-Catalytic Solid-Gas Reactions," *Ind. Eng. Chem. Process Design Develop.*, **10**, 164 (1971).
Johnson, J. L., "Kinetics of Bituminous Coal Char Gasification with Gases Containing Steam and Hydrogen," *Adv. in Chem. Ser. A.C.S.*, 131 (1974).
Rudolph, P., "Processing of American Coals in Lurgi Gasifier," Proceedings of 6th Synthetic Pipeline Gas Symposium, p. 269 (1974).
———, "The Lurgi Process. The Route to SNG From Coal," Proceedings of 4th Synthetic Pipeline Gas Symposium, p. 175 (1972).
Wen, C. Y., and S. C. Wang, "Thermal and Diffusional Effects in Non-Catalytic Solid-Gas Reactions," *Ind. Eng. Chem.*, **62**, No. 8, 30 (1970).
Woodmansee, D. E., "Modeling of Fixed Bed Gas Producer Performance," 5th Conference on Synthetic Fuels from Coal, Univ. Okla., Stillwater (May, 1975).
Yoon, H., James Wei, Morton M. Denn, "A Model for Moving Bed Coal Gasification Reactors," presented AIChE Annual Meeting, Chicago (1976).

Manuscript received June 29 and accepted August 23, 1977.

A Formulation for ϵ_M and ϵ_H Based on the Surface Renewal Principle

LINDON C. THOMAS

Department of Mechanical Engineering
University of Petroleum and Minerals
Dhahran, Saudi Arabia

This paper presents a simplified formulation for ϵ_H for low to moderate Prandtl number fluids and for ϵ_M that utilizes the surface renewal model, but which also accounts for the fact that eddies do not reach the surface. The predicted trend in Pr_t obtained on the basis of the present analysis suggests an opposite dependency on Pr to that predicted by many of the other analyses.

SCOPE

The primary objective of this paper is to demonstrate the relationship between the surface renewal approach to turbulence and the classical eddy diffusivity concept. The compatibility between these two approaches to turbulence has been previously demonstrated in the context of a comprehensive but complicated surface rejuvenation formulation. A simpler development is presented in this paper for moderate Prandtl numbers ($0.5 \lesssim Pr \lesssim 5.0$)

L. C. Thomas is presently on leave from the University of Akron, Akron, Ohio.

and for the wall region ($y^+ \lesssim 30$) that is based on the surface renewal model, but which also accounts for the fact that eddies do not generally reach the surface. Another objective of this study is to develop an alternative means of obtaining predictions for the turbulent Prandtl number Pr_t within the thermal wall region.

CONCLUSIONS AND SIGNIFICANCE

Although the surface renewal approach does not require the use of the eddy-viscosity or eddy-conductivity concepts, this paper demonstrates that predictions can be developed for ϵ_M and ϵ_H by use of this new approach to modeling wall turbulence together with a simplified treatment of the unreplenished layer of fluid at the wall. Importantly, this formulation retains the essential characteristics of the more comprehensive surface rejuvenation model for low to moderate Pr. Thus, a clearer picture of the relationship between the principle of surface renewal and the classical eddy diffusivity concept is provided. In addition to providing predictions for ϵ_M and ϵ_H which are in basic agreement with available experimental data within the wall region, the limiting predictions for ϵ_M as $y^+ \rightarrow 0$ are compatible with the well-known results of Elrod (1960) and Wasan et al. (1963). In contrast, earlier formulations (Thomas and Rajagopal, 1973; Hinze, 1959) which did not account for the unreplenished fluid at the wall led to a linear dependence of ϵ_M and ϵ_H on y^+ as $y^+ \rightarrow 0$.

In regard to the variation of Pr_t within the wall region with Pr, the present analysis suggests $Pr_t > 1$ for $Pr > 1$ and $Pr_t < 1$ for $Pr < 1$. Previous formulations by Na and Habib (1973), Jenkins (1951), Cebeci (1974), Rohsenow and Cohen (1960), and others lead to the opposite effect. Although insufficient data within the wall region are available to totally substantiate the predictions of the present or previous analyses, the recent data by McElgiot et al. (1976) for air and the data for high Pr (or Sc) fluids are in basic agreement with the variation of Pr_t, with Pr which is suggested by the present analysis. (Of course, use of the surface renewal principle in modeling heat or mass transfer for high Pr or Sc fluids requires the use of the more comprehensive surface rejuvenation model.)

Concerning the prediction $Pr_t < 1$ for $Pr < 1$, it should be emphasized that the present analysis only applies within the wall region. The analyses of Jenkins (1951) and Rohsenow and Cohen (1960) were developed primarily for the turbulent core. Indeed, these analyses have been found to provide reasonable predictions for Pr_t away from the wall for liquid metals (White, 1974), but they slightly overpredict Pr_t for air. On the basis of the present analysis and somewhat limited experimental data, it would appear that the use of these well-known formulations within the wall region is not appropriate. The fact that the present analysis gives rise to predictions for Pr_t vs. Pr that are in conflict with the wall region analyses by Na and Habib (1973) and Cebeci (1974) which are based on the popular van Driest damping factor approach should provide the motivation for further experimental and theoretical study of this problem.

Because of the nature of the surface renewal model, the primary weakness of the approach put forth in this paper is the physical limitation to predictions for the transport properties in question ($\overline{u}$, $\overline{T}$, ϵ_M and ϵ_H) to the wall region ($y^+ \lesssim 30$). Significantly and conversely, the primary weakness of the classical approach, aside from the somewhat heavy dependence upon indirect empirical input, is the uncertainty of the various semiempirical relationships for ϵ_M and ϵ_H (especially for high Pr) which are used for the wall region. Hence, the potential of great complement appears to exist between these two approaches.

The surface renewal model has been found to be particularly useful in the analysis of turbulent convection processes for low to moderate Prandtl number fluids ($Pr \lesssim 5.0$) (Einstein and Li, 1956; Hanratty, 1956; Meek and Baer 1972; Thomas, 1970). This simple model is based on the concept that elements of fluid intermittently move from the turbulent core into direct contact with the wall. Unsteady molecular transport is presumed to govern during the residency of fluid within the wall region. This model is in basic agreement with important characteristics of turbulent shear flow reported in visualization studies (Popovich and Hummel, 1967; Schraub et al., 1965), except for the fact that the inrush phase has not been found to bring fluid into direct contact with the wall.

It should be observed that the surface renewal approach treats turbulence as a time convective phenomenon, without eddy-viscosity or eddy-conductivity parameters. However, because traditional analyses of turbulence have been developed on the basis of the eddy diffusivity concept, the formulation of relationship for ϵ_M and ϵ_H based on the principle of surface renewal is desirable. A preliminary surface renewal based formulation for ϵ_M and ϵ_H which demonstrates the underlying concept has been developed (Thomas and Rajagopal, 1973). Predictions for ϵ_M based on this model are compared with empirical correlations in Figure 1. Because the elementary form of the surface renewal model was used in this formulation with no consideration given to the unreplenished fluid at the surface, the resulting predictions for ϵ_M and ϵ_H lie well above accepted correlations in the region close to the wall ($y^+ < 20$). The fact that this simple model leads to the inappropriate result $\epsilon_M \sim y^+$ as y^+ becomes small discouraged workers such as Hinze (1959) and others during the formative stage of this approach in the 1950's. This apparent incompatibility between the principle of surface renewal and the classical approach has been recently erased by a formulation for ϵ_M and ϵ_H obtained on the basis of the more representative surface rejuvenation model (Rajagopal, 1973; Rajagopal and Thomas, 1974). This model accounts for fluid elements not moving into direct contact with the surface.

Although the surface rejuvenation model provides a very comprehensive simulation of the actual turbulent transport mechanism, the mathematical complexities in-

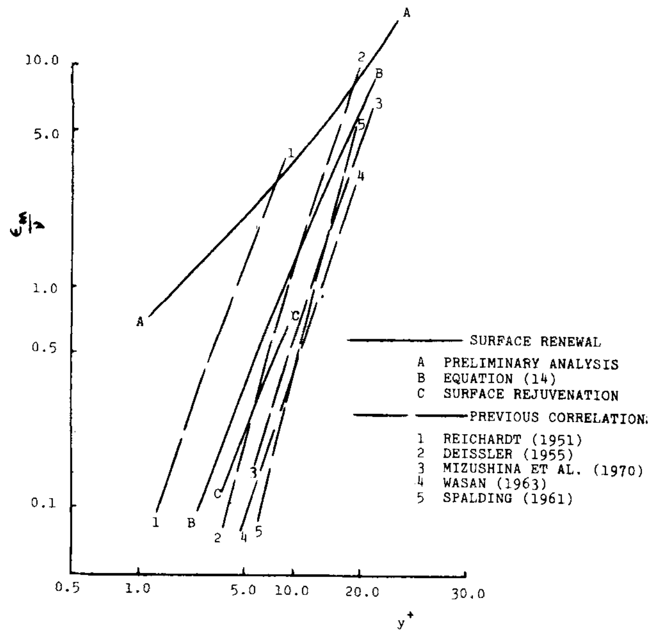

Fig. 1. Comparison of predictions for ϵ_M with previous correlations.

volved in this approach distract somewhat from the important issues involved in the appropriate application of the surface renewal concept. Therefore, a simpler formulation for ϵ_H for low to moderate Prandtl number fluids and for ϵ_M is now presented for the wall region which utilizes the surface renewal model but which also accounts for the fact that eddies do not reach the surface. Parenthetically, this simplified analysis is made possible by the fact that predictions for mean temperature profile $\bar{T}$ and heat transfer $\bar{q}_0$ (for low to moderate Pr) and mean velocity profile $\bar{u}$ are not too sensitive to the unreplenished layer of fluid at the surface. As such, predictions for $\bar{T}$, $\bar{q}_0$ and $\bar{u}$ based on both the surface renewal model and the surface rejuvenation model are essentially the same (Meek and Baer, 1972; Thomas et al., 1971; Thomas and Fan, 1971).

ANALYSIS

In the context of the surface rejuvenation concept, the energy per unit area carried across any given y plane e by an individual fluid element which has come to distance H ($<y$) is

$$e = \int_H^Y \rho c_p (T(\tau, \xi) - T_i)\, d\xi \tag{1}$$

where T_i represents the temperature of the fresh fluid brought into the wall region, and $T(\tau, y)$ is the temperature profile within the wall region at the instant proceeding rejuvenation; τ is the length of time an individual fluid element resides in the wall region. Therefore, an expression can be written for the average energy per unit area carried across the y plane by all eddies which move to within a distance H ($<y$) of the wall $\bar{e}$ as

$$\bar{e} = \int_H^Y \rho c_p (\bar{T}(\bar{\tau}, \xi) - T_i)\, d\xi \tag{2}$$

where $\bar{T}(\bar{\tau}, y)$ is the mean initial profile seen by incoming eddies (that is, mean-and-end residence time profile).

To account for the contribution to ϵ_H at y of all eddies that move to within distances less than y from the wall e_t, the approach distance distribution $P_H(H)$ is introduced to give (for constant properties)

$$e_t = \rho c_p \int_o^y P_H(H) \int_H^y (\bar{T}(\bar{\tau}, \xi) - T_i)\, d\xi\, dH \tag{3}$$

An expression now can be written for the mean rate of energy per unit area carried across the y plane by the transverse fluid motion associated with the surface rejuvenation process $\dot{e}_t$ of the form

$$\dot{e}_t = e_t/\bar{\bar{\tau}} \tag{4}$$

where $1/\bar{\bar{\tau}}$ represents the mean frequency of turbulent exchange below y. Note that since $\bar{\tau}$ designates the mean residence time within the entire wall region, $\bar{\bar{\tau}}$ is greater than $\bar{\tau}$. An expression can be written for $\bar{\bar{\tau}}$ of the form

$$\bar{\bar{\tau}} = \bar{A}\left[\int_o^y P_H(H)\, dH \right]^{-1} \tag{5}$$

where $1/\bar{A}$ is the mean frequency of the rejuvenation just outside the wall region.

ϵ_H and $\dot{e}_t$ are related by

$$\dot{e}_t = -\rho c_p \epsilon_H \frac{\partial \bar{T}}{\partial y} \tag{6}$$

such that an equation can be written for ϵ_H of the form

$$\epsilon_H = -\left\{ \int_o^y P_H(H) \int_H^y [\bar{T}(\bar{\tau}, \xi) - T_i]\, d\xi\, dH \right\} \left[\int_o^y P_H(H)\, dH \right] \left(\bar{A} \frac{\partial \bar{T}}{\partial y} \right)^{-1} \tag{7}$$

Because this relationship for $\dot{e}_t$ has been formulated directly from the surface rejuvenation concept, and ϵ_H is written by definition, this equation provides the interrelationship between the surface renewal and the classical approaches.

As mentioned, predictions for the mean transport properties obtained from the surface renewal model and the surface rejuvenation model are essentially the same for low to moderate values of Pr ($Pr < 5.0$). Therefore, expressions can be written for $\bar{T}(\bar{\tau}, y)$ and $\partial\bar{T}/\partial y$ by use of the elementary surface renewal model. An expression can be written for the mean temperature profile $\bar{T}$ on the basis of this model as (Thomas, 1970)

$$\frac{\bar{T} - T_i}{T_o - T_i} = \exp\left(-\frac{y}{\sqrt{\alpha \bar{\tau}}} \right) \tag{8}$$

for the random contact time distribution

$$\phi(\theta) = \frac{1}{\bar{\tau}} \exp\left(-\frac{\theta}{\bar{\tau}} \right) \tag{9}$$

Hence, $\partial\bar{T}/\partial y$ takes the form

$$\frac{\partial \bar{T}}{\partial y} = -\left(\frac{T_o - T_i}{\sqrt{\alpha \bar{\tau}}} \right) \exp\left(-\frac{y}{\sqrt{\alpha \bar{\tau}}} \right) \tag{10}$$

Because the random contact time and residence time distributions are identical, the mean initial temperature profile $\bar{T}(\bar{\tau}, y)$ can be set equal to the mean temperature profile $\bar{T}$.

On the basis of indirect experimental evidence in which the surface rejuvenation model has been found to correlate experimental heat transfer data for very high values of the Prandtl number (Thomas et al., 1971; Thomas and Fan, 1971), $P_H(H)$ has been taken as

$$P_H(H) = \frac{1}{\overline{H}} \exp\left(-\frac{H}{\overline{H}} \right) \tag{11}$$

H^+ has been set equal to 5 on the basis of experimental work by Popovich and Hummel (1967).

A relationship has been written for $\overline{\tau}$ on the basis of the surface renewal model of the form (for negligible axial pressure gradient and curvature effects) (Thomas, 1970)

$$U^* \sqrt{\frac{\overline{\tau}}{\nu}} = \frac{U_i}{U^*} \tag{12}$$

where U_i/U^* can be approximated by $U_i/U^* \simeq 15.97$ for fully turbulent flow. This expression has been found to be in good agreement with experimental measurements obtained by flush mounted anemometer probes (Meek and Baer, 1972; Thomas and Greene, 1973). Measurements also have been made for the mean burst period at $y^+ = 15$ by visualization techniques. Summaries of the results of these types of measurements are presented by Meek (1972) and Lafuer and Narayanan (1971). The measurements at $y^+ \simeq 15$ are approximately 2/3 of $\overline{\tau}$. Because these measurements were taken in the vicinity of the outer wall region, $\overline{A}/\overline{\tau}$ can be approximated by 2/3.

The substitution of the above expressions for $\partial\overline{T}/\partial y$, $\overline{T}(\overline{\tau}, y), \overline{\tau}$ and $P_H(H)$ into Equation (7), and the necessary integration yields

$$\frac{\epsilon_H}{\nu} = \frac{1}{Pr} \exp\left(\frac{y}{\sqrt{\alpha\overline{\tau}}} \right) \left[1 - \exp\left(-\frac{y}{\overline{H}} \right) \right]$$
$$\left\{ \left(1 + \frac{\overline{H}}{\sqrt{\alpha\overline{\tau}}} \right)^{-1} \left[1 - \exp\left(-\frac{y}{\overline{H}} - \frac{y}{\sqrt{\alpha\overline{\tau}}} \right) \right] \right.$$
$$\left. + \exp\left(-\frac{y}{\sqrt{\alpha\overline{\tau}}} \right) \left[\exp\left(-\frac{y}{\overline{H}} \right) - 1 \right] \right\} \frac{\overline{\tau}}{\overline{A}} \tag{13}$$

A similar formulation procedure gives rise to an expression for ϵ_m of the form

$$\frac{\epsilon_M}{\nu} = \exp\left(\frac{y}{\sqrt{\nu\overline{\tau}}} \right) \left[1 - \exp\left(-\frac{y}{\overline{H}} \right) \right]$$

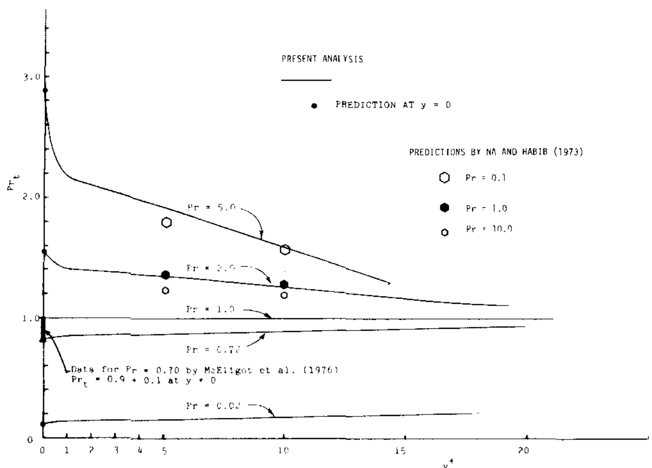

Fig. 2. Predictions for turbulent Prandtl number.

$$\left\{ \left(1 + \frac{\overline{H}}{\sqrt{\nu\overline{\tau}}} \right)^{-1} \left[1 - \exp\left(-\frac{y}{\overline{H}} - \frac{y}{\sqrt{\nu\overline{\tau}}} \right) \right] \right.$$
$$\left. + \exp\left(-\frac{y}{\sqrt{\nu\overline{\tau}}} \right) \left[\exp\left(-\frac{y}{\overline{H}} \right) - 1 \right] \right\} \frac{\overline{\tau}}{\overline{A}} \tag{14}$$

An expression can be obtained for the turbulent Prandtl number $Pr_t (\equiv \epsilon_M/\epsilon_H)$ by merely combining Equations (13) and (14). Note that Equations (13) and (14) express ϵ_M and ϵ_H in terms of the usual parameters y, ν, α, and the surface renewal modeling parameters $\overline{\tau}$ and $\overline{H}$.

RESULTS AND DISCUSSION

Consideration is first given to the limiting predictions for ϵ_M and ϵ_H as $y^+ \to 0$. As y^+ approaches zero, Equations (13) and (14) reduce to

$$\lim_{y^+ \to 0} \frac{\epsilon_H}{\nu} = \left(1 + \frac{\overline{H}}{\sqrt{\nu\overline{\tau}}} \sqrt{Pr} \right)^{-1} \frac{\psi}{\sqrt{Pr}} y^{+3} \tag{15}$$

$$\lim_{y^+ \to 0} \frac{\epsilon_M}{\nu} = \left(1 + \frac{\overline{H}}{\sqrt{\nu\overline{\tau}}} \right)^{-1} \psi y^{+3} \tag{16}$$

where $\psi = 2H^{+2}U^*\sqrt{\overline{\tau}/\nu}$. This functional dependency of ϵ_M and ϵ_H on y^+ very near the well is in basic agreement with the semitheoretical predictions of Elrod (1960) and Wasan et al. (1963). As indicated in the introduction, the failure to account for the unreplenished layer of fluid (see Thomas and Rajagopal, 1973) leads to an inappropriate linear dependence of ϵ_M and ϵ_H on y^+ as $y^+ \to 0$.

With $\overline{\tau}$ given by Equation (12), $\overline{A}/\overline{\tau} = 2/3$, and H^+ set equal to 5, Equations (13) and (14) have been evaluated for values of Pr between 0.02 and 6.0. The predictions for ϵ_M/ν are compared with several empirical correlations in Figure 1. In addition, predictions for ϵ_M based on the earlier preliminary surface renewal formulation (Thomas and Rajagopal, 1973) and the surface rejuvenation formulation (with $\overline{A}/\overline{\tau}$ set equal to unity) (Rajagopal, 1973; Thomas and Rajagopal, 1974) are shown in Figure 1. The present simplified analysis leads to predictions for ϵ_M which are in good agreement with calculations based on the more comprehensive surface rejuvenation model. The predictions for ϵ_M are also seen to be in reasonably good agreement with the limited empirical correlations of available wall region data.

Predictions for ϵ_H are shown in terms of the turbulent Prandtl number in Figure 2 as a function of Pr and y^+. Calculations based on Na and Habib's (1973) analysis are also shown. It should be mentioned that formulations for Pr_t by Jenkins (1951), Rohsenow and Cohen (1960), and Cebeci (1974) produce the same general variation with Pr. The predicted trend in Pr_t based on the surface renewal approach presented herein suggests a totally opposite dependency on Pr to that predicted by Na and Habib, Jenkins, Rohsenow and Cohen, and Cebeci.

The question regarding the dependence of Pr_t on Pr has been a point of controversy for some time because of the scarcity of reliable experimental data, especially within the wall region. As mentioned by Cebeci (1974), "the literature discusses the relationship between ϵ_M and ϵ_H at great length without definite conclusions." Based on an extensive review of data for Pr_t, Reynolds (1975) concluded that the experimental evidence was contradictory and only a very course filter for hypothesized models.

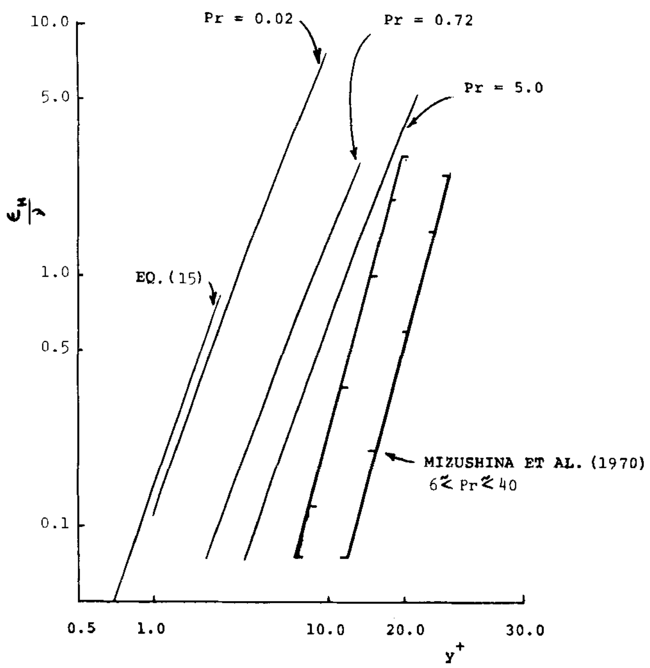

Fig. 3. Comparison of predictions for ϵ_H with data by Mizushina et al.

McEligot and co-workers (1976) were motivated by this lack of confidence in existing data to develop a new approach to measuring Pr_t at the wall. The results of their single experiment for air ($Pr \simeq 0.70$) is compared with the results of the present analysis in Figure 2. Incidently, McEligot et al. compared their experimental result with several analyses and concluded that most of these early analyses do not have a reasonable asymptote as the wall is approached.

A much more sensitive test for Pr_t occurs for turbulent convective heat or mass transfer for high Pr or Sc fluids. Because large thermal and concentration gradients occur very near the wall, predictions for the mean transport of heat and mass are quite noticeably influenced by assumptions for ϵ_H and ϵ_D near the wall. Recent measurements for ϵ_H have been reported by Mizushina et al. (1970) in the region $4 < y^+ < 100$ for $Pr = 6 \sim 40$. The envelope in which these data lie is shown in Figure 3. These data all lie below the generally accepted correlations for ϵ_M, such that $Pr_t > 1$. This result is consistent with the predictions of the present analysis. In a more recent study by Sherwood et al. (1968), the correlation of heat and mass transfer data of numerous investigators for very high Pr and Sc required the use of $Pr_t > 1$ and $Sc_t > 1$ which is in accordance with the present analysis. In fact, Sherwood et al. (1968) found that the use of the relationship $Pr_t = \sqrt{Pr}$ for $y^+ < 4$, which was arrived at on the basis of the simple surface renewal formulation with $\overline{H} = 0$, led to predictions for heat and mass transfer which were in good agreement with data.

NOTATION

$1/\overline{A}$ = mean frequency of rejuvenation just outside wall region
c_p = specific heat
D = molecular diffusivity of mass transfer
f = Fanning friction factor
H = approach distance
$\overline{H}$ = mean approach distance ($H^+ = \overline{H}U^*/\nu$)
Pr = molecular Prandtl number ($\equiv \nu/\alpha$)
Pr_t = turbulent Prandtl number ($\equiv \epsilon_m/\epsilon_H$)
$P_H(H)$ = approach distance distribution
Re = Reynolds number ($\equiv DU_b/\nu$)
Sc = molecular Schmidt number ($\equiv \nu/D$)
Sc_t = turbulent Schmidt number ($\equiv \epsilon_m/\epsilon_D$)
$T(\tau)$ = instantaneous end-residence time temperature profile
$\overline{T}(\overline{\tau})$ = mean end-residence time temperature profile
$\overline{T}$ = mean temperature profile
T_i = eddy temperature at first instant of renewal
U_i = eddy velocity at first instant of renewal
U^* = friction velocity ($\equiv \sqrt{\overline{\tau}_o/\rho}$)
y = distance from wall

Greek Letters

α = molecular thermal diffusivity
ν = kinematic viscosity
ϵ_m = eddy diffusivity for momentum
ϵ_H, ϵ_D = eddy diffusivity for heat, mass
ρ = density
θ = instantaneous contact time
τ = residence time
$\overline{\tau}$ = mean residence time
$\overline{\tau}_o$ = mean wall shear stress
$1/\overline{\overline{\tau}}$ = mean frequency of turbulent exchange below y
ϕ = contact time distribution
ψ = $\equiv 2H^{+2}\,U^*\sqrt{\overline{\tau}/\nu}$

LITERATURE CITED

Cebeci, T., and A. M. O. Smith, *Analysis of Turbulent Boundary Layers*, Academic Press, New York (1974).
Cohen, L. S., M. S. thesis, Department of Nuclear Engineering, Mass. Inst. Technol., Cambridge (1960).
Corino, E. R., and R. S. Broadkey, "A Visual Investigation of the Wall Region in Turbulent Flow," *J. Fluid Mech.*, **37**, 1 (1969).
Deissler, R. G., *NASA Tech. Rept. 1210* (1955).
Einstein, H. A., and H. Li, 'The Viscous Sublayer Along a Smooth Boundary," *ASCE., J. Eng. Mech. Div.*, **82**, 293 (1956).
Elrod, H. G., "Note on the Turbulent Shear Stress Near a Wall," *J. Aero. Sci.*, 468 (1957); erratum, **27**, 145 (1960).
Hanratty, T. J., "Turbulent Exchange of Mass and Momentum With A Boundary," *AIChE J.*, **2**, 359 (1956).
Hinze, J. L., *Turbulence*, McGraw-Hill, New York (1959).
Jenkins, R., "Variation of the Eddy Conductivity with Prandtl Modulus and its Use in Predictions of Turbulent Heat Transfer Coefficients," Heat Transfer Fluid Mech. Inst., Stanford Univ. Press, Stanford, Calif., 147 (1951).
Lafuer, J., and M. A. Badri Narayanan, "Mean Period of the Turbulent Production Mechanism in a Boundary Layer," *J. Fluid Mech.*, **50**, 133 (1971).
McEligot, D. M., P. E. Pickett, and M. F. Tayler, "Measurement of Wall Region Turbulent Prandtl Numbers in Small Tubes," *Int. J. Heat Mass Transfer*, **19**, 799 (1976).
Meek, R. L., "Mean Period of Fluctuation Near the Wall in Turbulent Flow," *AIChE J.*, **18**, 854 (1972).
———, and A. D. Baer, "The Periodic Viscous Sublayer in Turbulent Flow," *ibid.*, **16**, 841 (1970).
Mizushina, T., R. Ito, and F. Oyino, "Eddy Diffusivity Distribution Near the Wall," 4th Int. Heat Transfer Conference FC2, 8 (1970).
Na, T. Y., and I. S. Habib, "Heat Transfer in Turbulent Pipe Flow Based on a New Mixing Length Model," *Appl. Sci. Res.*, **28**, 303 (1973).
Popovich, A. T., and R. L. Hummel, "Experimental Study of the Viscous Sublayer in Turbulent Pipe Flow," *AIChE J.*, **13**, 754 (1967).
Rajagopal, R., Ph.D. thesis, "Adaptation of Surface Renewal and Penetration Type Models to Turbulent Momentum and Heat Transfer for Newtonian and Non-Newtonian Fluids," Univ. Akron, Ohio (Dec., 1973).

———, and L. C. Thomas, "The Formulation of Relationships for ϵ_M and ϵ_H Based on a Physically Realistic Model of Turbulence," Int. Heat Transfer Conf., Tokyo, Japan (1974).
Reichardt, H., "Vollstandige Darstellung der Turbulenten Geschwin Geschwindigkeitsvereilung in glatten Leitungen," *Z. Angew. Math. Mech.*, 31, 208 (1951).
Schraub, F. A., J. Henry, R. W. Runstadler, and A. Little, "Use of Hydrogen Bubbles of Quantitative Determination of Time-Dependent Velocity Fields in Low Speed Water Flows," *J. Basic Eng., Trans ASME*, 87, 429 (1965).
Sherwood, T. K., K. A. Smith, and P. E. Fowler, "The Velocity and Eddy Viscosity Distribution in the Wall Region of Turbulent Pipe Flow," *Chem. Eng. Sci.*, 23, 1225 (1968).
Spalding, D. B., "A Single Formula for the "Law of the Wall", *J. Appl. Mech.*, 28, 455 (1961).
Thomas, L. C., "Temperature Profiles for Liquid Metals and Moderate-Prandtl Number Fluids," *J. Heat Transfer, Trans. ASME*, 92, 565 (1970).
———, B. T. F. Chung, and S. K. Mahaldar, "Temperature Profiles for Turbulent Flow of High Prandtl Number Fluids," *Int. J. Heat and Mass Transfer*, 14, 1665 (1971).
Thomas, L. C., and L. T. Fan, "Adaptation of the Surface Rejuvenation Model to Turbulent Heat and Mass Transfer at a Solid-Fluid Interface," *Ind. Eng. Chem. Fundamentals*, 10, 135 (1971).
Thomas, L. C., and H. L. Greene, "An Experimental and Theoretical Study of the Viscous Sublayer for Turbulent Tube Flow," Symposium on Turbulence in Liquids, The University of Missouri, Rolla (1973).
Thomas, L. C., and R. Rajagopal, "A Preliminary Formulation for ϵ_M and ϵ_H Based on the Elementary Surface Renewal and Penetration Model," A.I.Ch.E. Meeting, Philadelphia (1973).
Wasan, A. T., C. L. Tien, and C. R. Wilke, "Theoretical Correlation of Velocity and Eddy Viscosity for Flow Close to a Pipe Wall," *AIChE J.*, 9, 567 (1963).
White, F. M., *Viscous Fluid Flow*, McGraw-Hill, New York (1974).

Manuscript received October 6, 1975; revision received June 6, and accepted August 10, 1977.

Bubble Nucleation in Viscous Material Due to Gas Formation by a Chemical Reaction: Application to Coal Pyrolysis

AMIR ATTAR
Chemical Engineering Department
University of Houston
Houston, Texas 77004

Most coals swell when heated, and their plastic properties change. These phenomena are explained qualitatively and semiquantitatively using a theory which combines classical nucleation theory and chemical kinetics.

Classical bubble nucleation theory is modified to include cases where gases are formed in a liquid as a result of a chemical reaction. Part of the gas that is formed in the liquid escapes through the surface. However, the concentration of gas that remains in the solution increases until a critical concentration is reached. When the critical concentration of gas in the solution is reached, bubbles will begin to nucleate.

The value of the rate of nucleation has a critical upper limit which is determined by the thermodynamic properties of the solution. However, the kinetics of the heat, momentum, or mass transfer may reduce the thermodynamic rate.

The criteria to decide which transport mode limits the kinetics of the nucleation were derived and applied to melting coal. In coal, the rate of bubble nucleation appears to be limited by the rate of momentum transfer in the melt.

The size of the melting coal particle determines the ratio of surface area to volume and thus affects the kinetics of the accumulation of gaseous reaction product in the melt. Thus, smaller rates of bubble nucleation will be observed in smaller coal particles (very viscous melts).

Kinetic equations are derived and used to estimate the time that is required for bubbles to appear inside a coal particle. A general method is proposed to calculate the time-temperature contour at which nucleation will occur. The effect of the particle size on the time of nucleation is calculated. The results of the calculations yield values which compare very well with experimental results on coal.

SCOPE

The rate of bubble nucleation in coal due to the formation of gaseous reaction products plays a critical role in the pyrolysis process. The bubbles transport volatiles from the interior of the coal particle and serve as microreactors for secondary chemical reactions. Thus, the bubble nucleation process determines to a large extent the products and yields of pyrolysis reactions.

The theory of bubble nucleation in pure boiling liquid was developed by Volmer and Weber (1926), Farkas (1927) Becker and Doring (1951), Zeldovich (1943), and Frenkel (1943). Modifications of the theory to include the effect of the rate of heat transfer and the viscosity of the liquid were done by Kagan (1960). Blander et al. (1971) studied bubble nucleation in liquid mixtures. The theories of homogeneous and heterogeneous nucleation were reviewed by Blander and Katz (1975) and by Cole (1974).

The author is not aware of any experimental studies of bubble nucleation due to a chemical reaction in viscous